Short Papers

Computation of Propagation Constants for the Fundamental and Higher Order Modes in Microstrip

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Abstract—A method used to treat static problems in microstrip is extended to treat time-harmonic problems of covered and uncovered microstrip. Both longitudinal and transverse currents are taken into account. Impedance functions (integrals of Green's functions) for covered and uncovered microstrip are derived in terms of improper integrals (limits $0 \to \infty$). Accurate evaluation of these integrals is carried out. Matrix methods are then used to obtain propagation constants for the fundamental and higher order modes. Data obtained agree closely with experiment.

I. Introduction

For microstrip of commonly used dimensions, an electrostatic analysis is reasonably adequate at frequencies below S band. At frequencies above S band, especially for wide ($W/H \geq 0.5$) center conductors, microstrip is dispersive and departs significantly from the behavior predicted by static analysis. A number of attempts have been made recently to obtain time-harmonic solutions for microstrip. The majority of the authors have advanced solutions for the dominant mode of shielded microstrip [1]–[11]. Several authors have used the method of finite differences to analyze the higher order modes in shielded microstrip [8], [9]. The problem of open microstrip has been treated by several authors who have assumed particular current distributions [12], [13]. Getsinger [14] has used a ridge-guide approximation to microstrip and has obtained theoretical as well as experimental data.

In this short paper, the propagation constants for the dominant and the higher modes of open microstrip are computed. Both longitudinal and transverse currents are treated. The method of solution used is an extension of the Fourier integral method used previously by the authors to treat static problems of covered and multilayer microstrip [15], [16]. The Fourier integral (k-space) method is used in conjunction with the method of moments [17]. Pulse expansion functions are used for both longitudinal and transverse currents. A multiple-impulse weighting function is used, in a "quasi-Galerkin" solution. Propagation $e^{-jk_z z}$ is assumed and a characteristic (eigenvalue) equation is obtained for the propagation constants of the microstrip modes. The corresponding solution for covered microstrip is also given. Solutions to the characteristic equation are obtained using a computer program and the results are compared with available data.

II. The General Formulation for the Characteristic Equation

Fig. 1 shows the basic geometry considered for open (uncovered) microstrip. It is assumed that all quantities vary as $e^{-ik_z z}$. The allowed values of k_z are sought.

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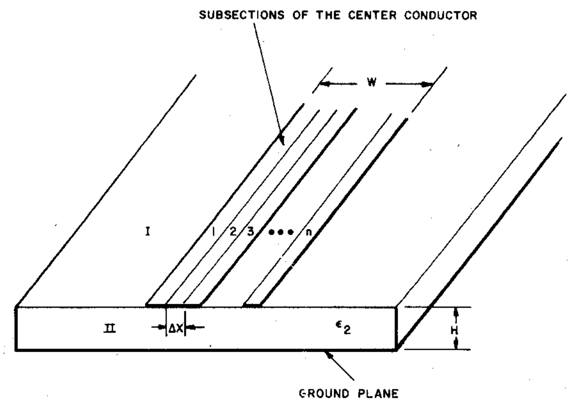

Fig. 1. Cross section of open (uncovered) microstrip.

The center conductor consists of n subsections each of width Δx. A typical subsection x_i ($i = 1,3,2,\cdots,n$) is traversed by both longitudinal (J_{zi}) and transverse (J_{xi}) current. The current densities on the center conductor may be written as

$$J_x = \sum_{i=1}^{n} J_{xi} f_{xi} e^{-jk_z z} \quad (1)$$

$$J_z = \sum_{i=1}^{n} J_{zi} f_{zi} e^{-ik_z z} \quad (2)$$

where J_{xi} and J_{zi} are unknown complex constants and where J_{zi} and f_{xi} are expansion functions [16]. An example is the pulse expansion function defined below

$$f_{xi} = f_{zi} = \begin{cases} 1, & \text{on } \Delta x_i \\ 0, & \text{on all other } \Delta x_i. \end{cases} \quad (3)$$

The tangential components of electric-field intensities on subsection i due the currents on subsection j are

$$e_{xi} = \sum_{j=1}^{n} J_{xj} \int_{\Delta x_j} f_{xi}(x') G_{xx}(\rho \mid \rho')\, dx' + J_{zj} \int_{\Delta x_j} f_{zj}(x') G_{xz}(\rho \mid \rho')\, dx' \quad (4)$$

and

$$e_{zi} = \sum_{j=1}^{n} J_{zj} \int_{\Delta x_j} f_{zi}(x') G_{zx}(\rho \mid \rho')\, dx' + J_{xj} \int_{\Delta x_j} f_{xj}(x') G_{zz}(\rho \mid \rho')\, dx' \quad (5)$$

where $G_{xz}(\rho \mid \rho')$ is, for instance, the tangential component of electric field at ρ due to a current J_{zj} at source point ρ'. Define a set of weighting functions $w_i(x)$ [17]. Multiply both sides of (4) and (5) by w_i and then integrate over the center conductor. Consequently, the right-hand sides of (4) and (5) yield E_{xi} and E_{zi}, respectively, which are weighted averages of the electric field over the length Δx_j. And the left-hand sides of (4) and (5) represent $Z_{xx}^{(ij)}$, $Z_{xz}^{(ij)}$, etc. [18]. The quantities $Z_{xx}^{(ij)}$, $Z_{xz}^{(ij)}$, etc., are "generalized" impedance functions for the problem. The relationship between the tangential components of the electric field and the currents for the n subsections may thus be

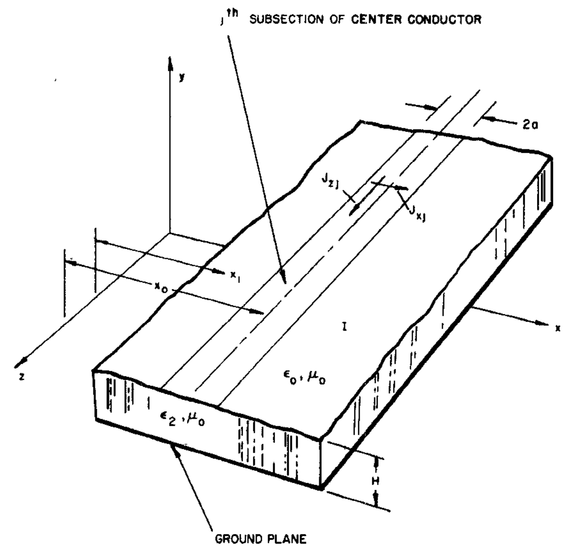

Fig. 2. Impedance-function geometry for uncovered microstrip ($2a \ll W$).

written in matrix form

$$\begin{bmatrix} E_x \\ E_z \end{bmatrix} = \begin{bmatrix} Z_{xz} & Z_{xx} \\ Z_{zz} & Z_{zx} \end{bmatrix} \begin{bmatrix} J_z \\ J_x \end{bmatrix} \tag{6}$$

where bold face denotes a matrix. The representation in (6) is of the familiar method-of-moments [17] form. Since the tangential components of the electric fields at the surface of the conductor vanish, (6) becomes

$$[Z(k_z)][J] = 0 \tag{7}$$

where $[Z]$ is a square matrix of order $2n$ (the generalized impedance matrix) and $[J]$ is a column matrix of the unknown currents. The characteristic equation is found by setting the determinant of $[Z]$ equal to zero

$$\det\,[Z(k_z)] = 0. \tag{8}$$

The proper values of K_z (normalized) for which (8) is satisfied are sought. The eigenvectors J_x, J_z may be obtained by solving (7) for a particular K_z.

The preceding analysis applies to the covered microstrip of Fig. 5 as well as the uncovered microstrip of Fig. 1, except that the Green's functions differ. The formulation can also be applied to multiple-conductor problems.

In the following analysis pulse expansion functions and multiple-impulse weighting functions are used, in a "quasi-Galerkin" solution.

III. Analysis

A. Impedance Functions for Uncovered Microstrip

In this section, the fields due to a subsection carrying longitudinal (J_{zj}) and transverse (J_{xj}) currents are determined. Fig. 2 shows a typical current-carrying subsection j. It is assumed that the currents are uniform over the subsection (pulse-expansion functions). The tangential components of the electric and magnetic fields in the plane of the interface are required. The problem is formulated in terms of fields TE to y and TM to y. This combination constitutes a complete set describing all possible modes for the geometry of Fig. 1. The solution starts with the proper selection of wave functions.

$$\psi = \int_{k_x}\int_{k_y} f(k_x,k_y) e^{jk_x x} e^{jk_y y} e^{-jk_z z}\, dk_x\, dk_y \tag{9}$$

where $f(k_x,k_y)$ is determined by applying the boundary conditions of the problem. The parameters k_x, k_y, and k_z in (12) are separation constants which satisfy

$$k_x^2 + k_y^2 + k_z^2 = k^2. \tag{10}$$

To formulate the problem in terms of fields TM and TE to y, let

$$A = \psi^M \hat{y} \tag{11}$$

$$F = \psi^E \hat{y} \tag{12}$$

where A and F are magnetic and electric vector potentials as defined by Harrington [19] and the superscripts M and E denote modes TM and TE to y, respectively. The electromagnetic field (in a source-free region) in terms of ψ^M and ψ^E is given by

$$E_x = \frac{1}{\hat{Y}}\frac{\partial \psi^M}{\partial x\,\partial y} + \frac{\partial \psi^E}{\partial z} \qquad H_x = -\frac{\partial \psi^M}{\partial z} + \frac{1}{\hat{Z}}\frac{\partial^2 \psi^E}{\partial x\,\partial y}$$

$$E_z = \frac{1}{\hat{Y}}\frac{\partial^2 \psi^M}{\partial z\,\partial y} - \frac{\partial \psi^E}{\partial x} \qquad H_z = \frac{\partial \psi^M}{\partial x} + \frac{1}{\hat{Z}}\frac{\partial^2 \psi^E}{\partial z\,\partial y} \tag{13}$$

where $\hat{Y} = j\omega\varepsilon$ and $\hat{Z} = j\omega\mu$.

For convenience, wave functions ψ^M and ψ^E satisfying the boundary conditions on tangential $\boldsymbol{E}$ at the ground plane ($Y = 0$) are selected

$$\psi_1^M = e^{-jk_z z}\int_{-\infty}^{\infty} A(k_x) e^{jk_{y1}(y-H)} e^{jk_x x}\, dk_x \tag{14}$$

$$\psi_2^M = e^{-jk_z z}\int_{-\infty}^{\infty} B(k_x)\frac{\cos k_{y2} y}{\sin k_{y2}{}^H} e^{jk_x x}\, dk_x \tag{15}$$

$$\psi_1^E = e^{-jk_z z}\int_{-\infty}^{\infty} C(k_x) e^{jk_{y1}(y-H)} e^{jk_x x}\, dk_x \tag{16}$$

$$\psi_2^E = e^{-jk_z z}\int_{-\infty}^{\infty} D(k_x)\frac{\sin k_{y2} y}{\sin k_{y2}{}^H} e^{jk_x x}\, dk_x. \tag{17}$$

The integration variable k_x (rather than k_y) is dictated by pulse representation used for the current. Subscripts 1 and 2 in (14)–(17) refer to regions I and II of Fig. 2 and the superscripts M and E refer to the wave functions for TM and TE, respectively. Quantities k_x and k_z are identical for both regions in order to satisfy boundary conditions at the interface ($y = H$).

Functions A, B, C, and D are determined by applying the following boundary conditions at the interface:

$$E_{x1} = E_{x2} \qquad H_{x2} - H_{x1} = J_{zj}$$

$$E_{z1} = E_{z2} \qquad H_{z1} - H_{z2} = J_{xj}$$

where J_{zj} and J_{xj} are the current densities [18]. Substitute functions A, B, C, and D into the wave functions in (14)–(17). The tangential components of electric fields may be obtained by substituting these wave functions into (13). The tangential components of the fields at $y = H$ are

$$E_x = \int_0^\infty [f(k_x)I_{zj} + g(k_x)I_{xj}]\, dk_x \tag{18}$$

$$E_z = \int_0^\infty [q(k_x)I_{zj} + f(k_x)I_{xj}]\, dk_x \tag{19}$$

where

$$f(k_x) = -\frac{2e^{-jk_z z}}{\pi \hat{Y}_1} \frac{k_z(k_{y2} + jTk_{y1}) \sin k_x a \sin k_x(x - x_0)}{(k_{y2} + j\varepsilon_r T k_{y1})(k_{y1} + jTk_{y2})}$$

$$g(k_x) = \frac{2je^{-jk_z z}}{\pi \hat{Y}_1} \frac{k_{y2}(k_1^2 - k_x^2) + jTk_{y1}(k_2^2 - k_x^2)}{k_x(k_{y2} + j\varepsilon_r T k_{y1})(k_{y1} + jTk_{y2})} \cdot \sin k_x a \cos k_x(x - x_0)$$

$$q(k_x) = \frac{2je^{-jk_z z}}{\pi \hat{Y}_1} \frac{k_{y2}(k_1^2 - k_z^2) + jTk_{y1}(k_2^2 - k_z^2)}{k_x(k_{y2} + j\varepsilon_r T k_{y1})(k_{y1} + jTk_{y2})} \cdot \sin k_x a \cos k_x(x - x_0) \tag{20}$$

where $T = \cot k_{y2}{}^H$. The range of integration is reduced to $(0 \to \infty)$ by using only the even parts of the integrands.

The improper integrals in (18) and (19) are evaluated by breaking into two integrals with ranges $(0 \to u)$ and $(u \to \infty)$. The first is evaluated by numerical integration using Simpson's rule. No poles in this region are encountered. The second is evaluated in closed form by 1) using large argument expressions of the integrand, 2) expanding in a Taylor series, and 3) integrating term by term. The details of this procedure are as follows.

Equations (18) and (19) may be written as

$$E_x = \int_0^u (fI_{zj} + gI_{xj})\, dk_x + \int_u^\infty (fI_{zj} + gI_{xj})\, dk_x \tag{21}$$

$$E_z = \int_0^u (qI_{zj} + gI_{xj})\, dk_x + \int_u^\infty (qI_{zj} + fI_{xj})\, dk_x. \tag{22}$$

The variable u in (21) and (22) is selected such that

$$T = \cot k_{y2}{}^H = j \quad \text{for} \quad |K_{y2}{}^H| \geq 10. \tag{23}$$

The integrands of the second integrals in (21) and (22) are expanded in a Taylor series in k_x after substituting (23) into (20). An expansion in k_{y1} or k_{y2} could also be used but is less convenient for this problem.

Next, write (21) and (22) as follows:

$$E_x = \int_0^u (fI_{zj} + gI_{xj})\, dk_x + E_x' \quad \text{(large argument part of } E_x)$$

$$E_z = \int_0^u (fI_{zj} + gI_{xj})\, dk_x + E_z' \quad \text{(large argument part of } E_z)$$

where

$$E_z' = \Delta \int_u^\infty \left\{ \left[\frac{1}{(E + F)K_x} - \frac{K_z^2}{(F + \varepsilon_r E)K_x} \right] \times I_{zj}(\sin QK_x + \sin Q'K_x) + \frac{jK_z(\cos Q'K_x - \cos QK_x)}{F + \varepsilon_r E} I_{xj} \right\} dK_x \tag{24}$$

$$E_x' = \Delta \int_u^\infty \left\{ \frac{jK_z(\cos Q'K_x - \cos QK_x)}{F + \varepsilon_r E} I_{zj} + \left[\frac{1}{(E + F)\, K_x} - \frac{K_x}{F + \varepsilon_r E} \right] I_{xj} \times (\sin QK_x + \sin Q'K_x) \right\} dK_x \tag{25}$$

where

$$\Delta = e^{-jk_z z}/\pi \hat{Y}_1, \qquad Q = (x - x_0)k_0 + ak_0$$

$$E = \sqrt{K_x^2 + K_z^2 - 1}, \qquad Q' = (x - x_0)k_0 - ak_0$$

$$F = \sqrt{K_x^2 + K_z^2 - \varepsilon_r}, \qquad K_x = k_x/k_0,\ K_z = k_z/k_0.$$

E, F should not be confused with vectors $\boldsymbol{E}$, $\boldsymbol{F}$.

Rewrite (24) and (25) as follows:

$$E_z' = (\mathrm{I} - K_z^2 \mathrm{II})I_{zj} + jK_z \mathrm{IV} I_{xj}$$

$$E_x' = jK_z \mathrm{IV} I_{zj} + (\mathrm{II} - \mathrm{III})I_{xj} \tag{26}$$

where

$$\mathrm{I} = \int_u^\infty \frac{\sin QK_x + \sin Q'K_x}{(E + F)K_x}\, dK_x \tag{27}$$

$$\mathrm{II} = \int_u^\infty \frac{\sin QK_x + \sin Q'K_x}{(F + \varepsilon_r E)K_x}\, dK_x \tag{28}$$

$$\mathrm{III} = \int_u^\infty \frac{K_x(\sin QK_x + \sin Q'K_x)}{F + \varepsilon_r E}\, dK_x \tag{29}$$

$$\mathrm{IV} = \int_u^\infty \frac{\cos Q'K_x - \cos QK_x}{F + \varepsilon_r E}\, dK_x. \tag{30}$$

To carry out the integration in (27)–(30) the expansions for the expressions $1/(E + F)$ and $1/(F + \varepsilon E)$ are needed. For brevity, only the integral in (27) will be considered here. The evaluation of the integrals in (28)–(30) may be carried out in a similar manner. Expand the expression $1/(E + F)$ in powers of K.

$$\frac{1}{E + F} = \frac{1}{\varepsilon_r - 1} \sum_{n=1}^\infty C_n \frac{1}{K_x^{2n}}. \tag{31}$$

Substituting (31) into (27) we obtain

$$\mathrm{I} = \frac{1}{\varepsilon_r - 1} \sum_{n=1}^\infty C_n \int_u^\infty \left(\frac{\sin QK_x}{K_x^{2n}} + \frac{\sin QK_x}{K_x^{2n}} \right) dK_x. \tag{32}$$

Using the appropriate integration formula [20] we obtain the final result for I as follows [18]:

$$\mathrm{I} = \frac{1}{\alpha^2 - \beta^2} \sum_{n=1}^\infty C_n' \frac{(-1)^{n+1}}{(2n - 1)!} \left\{ \sum_{k=0}^{n-2} (-1)^k (2k + 1)! \cdot \left(\frac{\cos x_1}{x_1^{2k+2}} + \frac{\cos x_2}{x_2^{2k+2}} \right) \right.$$

$$+ \sum_{k=0}^{n-1} (-1)^{k+1}(2k!) \frac{\sin x_1}{x_1^{2k+1}} + \frac{\sin x_2}{x_2^{2k+1}}$$

$$+ \frac{ci(x_1)}{x_1} + \frac{ci(x_2)}{x_2} \Bigg\} (Q^{2n-1} + Q'^{2n-1})$$

where

$$ci = \text{the cosine integral}$$

$$x_1 = uQ$$

$$x_2 = uQ'.$$

Finally,

$$E_x^{(ij)} = \int_0^u (fJ_{zj} + gI_{xj})\, dK_y + jK_z I_{zj}\mathrm{IV} + (\mathrm{II} - \mathrm{III})I_{xj} \quad (33)$$

$$E_z^{(ij)} = \int_0^u (qI_{zj} + fI_{xj})\, dK_y + (\mathrm{I} - K_z\mathrm{II})I_{zj} + jK_z I_{xj}\mathrm{IV}. \quad (34)$$

Equations (33) and (34) are thus the final expressions for the tangential electric fields at field point x_i on the interface (Fig. 2) due to a current-carrying subsection centered at source point x_j.

B. Impedance Functions for Covered Microstrip

The impedance function for covered microstrip may be derived by a procedure similar to that described in the previous subsection, except that it is necessary to choose ψ functions which satisfy the boundary conditions on tangential electric field at the top cover ($y = E$) (see Fig. 3). The results are as follows:

$$E_x^{(ij)} = \int_0^\infty r(k_x)I_{zj} + s(k_x)I_{xj}\, dk_x \quad (35)$$

$$E_z^{(ij)} = \int_0^\infty t(k_x)I_{zj} + r(k_x)I_{xj}\, dk_x \quad (36)$$

where

$$r(k_x) = \frac{2je^{-jk_z z}}{\pi \hat{y}_1} \frac{k_z(k_{y2}T_1 + k_{y1}T_2) \sin k_x a \sin k_x(x - x_0)}{(k_{y2}T_1 + \varepsilon_r k_{y1}T_2)(k_{y1}T_1 + k_{y2}T_2)}$$

$$s(k_x) = \frac{2e^{-jk_z z}}{\pi \hat{y}_1} \frac{k_{y2}T_1(k_1^2 - k_x^2) + k_{y1}T_2(k_2^2 - k_x^2)}{k_x(k_{y2}T_1 + \varepsilon_r k_{y1}T_2)(k_{y1}T_1 + k_{y2}T_2)} \cdot \sin k_x a \cos k_y(x - x_0)$$

$$t(k_x) = \frac{2e^{-jk_z z}}{\pi \hat{y}_1} \frac{k_{y2}T_1(k_1^2 - k_z^2) + k_{y1}T_2(k_2^2 - k_z^2)}{k_x(k_{y2}T_1 + \varepsilon_r k_{y1}T_2)(k_{y1}T_1 + k_{y2}T_2)} \cdot \sin k_x a \cos k_x(x - x_0)$$

and where $T_1 = \cot k_{y1}(E - H)$ and $T_2 = \cot k_{y2}H$. Note that if the top cover is removed (let $E \to \infty$), (34) and (35) will be reduced to (18) and (19). Equations (34) and (35) may be evaluated in a manner similar to that previously described for (18) and (19).

IV. Computational Procedure for Propagation Constants

Once the impedance functions are obtained, substitution in (8) yields the desired eigenvalue equation. The procedure for the solution of this equation is as follows.

For a particular choice of n, the number of subsections (see Fig. 1) there are a finite number of K_z for which the determinant is zero. These roots are found by plotting the determinant versus K_z. There are extraneous roots which are improper values of K_z [16].

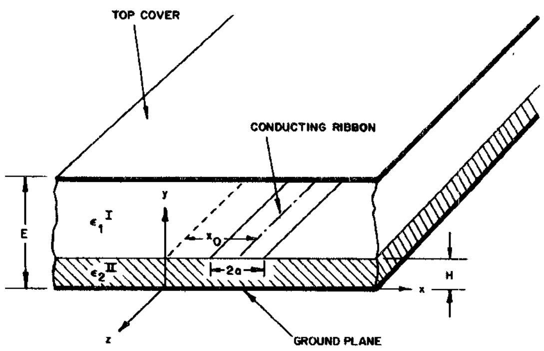

Fig. 3. Impedance-function geometry for covered microstrip.

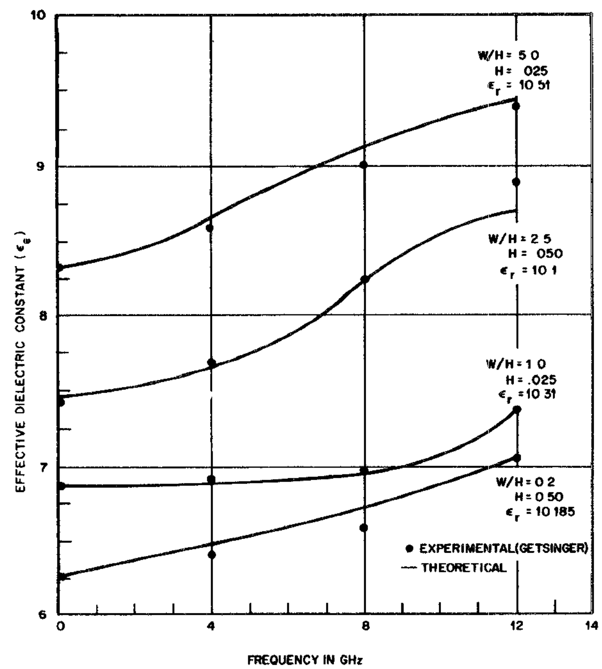

Fig. 4. Effective dielectric constant for fundamental mode in microstrip.

The extraneous roots are found by varying both frequency and the number of subsections. The proper roots are not significantly affected, whereas the extraneous roots are changed radically. This combined criterion is found to be very effective in detecting extraneous roots.

A computer program in Fortran IV was prepared to calculate the eigenvalues of K_z for the fundamental and higher order modes in microstrip. The results for the dominant mode are plotted in Fig. 4 (the ordinate is ε_e where $\varepsilon_e = (k_z/k_0)^2$). Note the good agreement with experimental data. The maximum deviation is 4 percent in ε_e, which corresponds to 2 percent in K_z. Note also that the static (dispersionless) solution would be represented by a horizontal line. Fig. 5 shows some results for dominant and higher order modes as compared with theoretical results by Van de Capelle and Lypaert [12] and Getsinger [14]. The maximum deviation is 4 percent in ε_e or 2 percent in K_z. The computation time for any one of the curves in Fig. 5 was approximately 20 min using the Honeywell 635. This includes the time necessary for generating extra data in order to select the correct values of K_z.

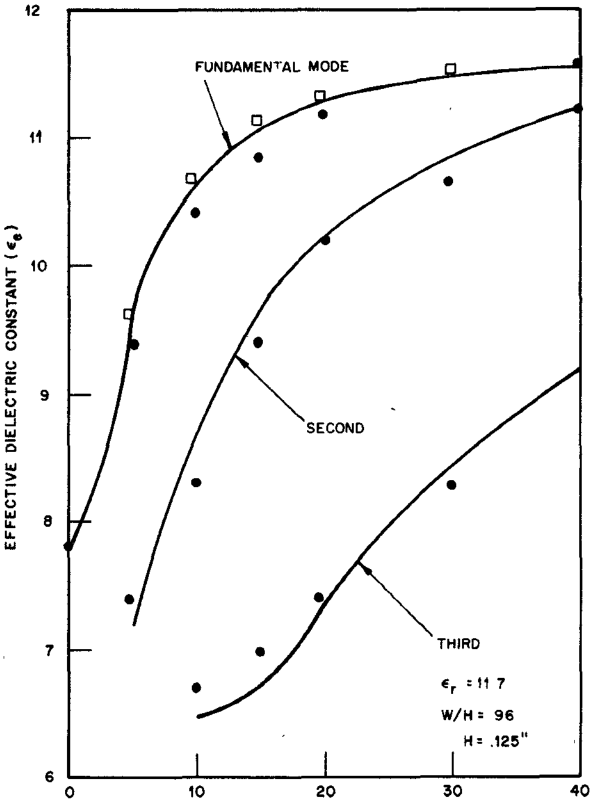

Fig. 5. Normalized propagation constant for fundamental and higher order modes in open microstrip (● Van de Capelle and Lypaert [12], □ Getsinger [14]).

V. Conclusions

A continuous-spectrum method has been used in conjunction with the method of moments to treat the time-harmonic solution of covered and uncovered microstrips. Both longitudinal and transverse currents were considered in the analysis. The propagation constants for fundamental and higher order modes in open microstrip were calculated. In each case the results are in good agreement with available theoretical and experimental data. The results are accurate to within 4 percent.

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General Noise Analysis for Bias- and RF-Voltage-Dependent Transferred-Electron Devices

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***Abstract*—The general AM and FM noise spectrum analysis of Sweet for transferred-electron devices is extended to include the variation of device admittance with both bias- and RF-voltage amplitudes. This is important because recent investigations by the author suggest that there are significant variations of device admittance with both parameters. Also the expressions for the AM and FM noise spectra are formulated in terms of the more basic quantities such as stored charge, modulation sensitivities, and incremental admittance.**

Introduction

The lumped-circuit analysis of noise in self-excited oscillators has received considerable attention. Edson [1], Mullen [2], and van der Pol [3] wrote basic papers on this subject. As different self-oscillating devices have been developed, their noise properties have been studied in detail. Lax [4] underscored this individuality of self-excited oscillators when he observed that "the noise mixes with the signal in a complex fashion that is quite different from ordinary nonlinear systems .... It is not satisfactory to represent the spectrum as a delta function signal plus a background. The noise will spread the delta function spectrum into a finite width." This complex mixing is dependent both on the device properties and on the device environment. Hence, it is necessary to combine an oscillator device model with an RF-circuit model to completely study oscillator noise properties. This short paper extends the theoretical groundwork for the general noise analysis of transferred-electron (TE) devices.

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